

# Optimal hot rolling production scheduling for economic load dispatch under time-of-use electricity pricing

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Received: date / Accepted: date

**Abstract** Time-of-Use (TOU) electricity pricing provides a new opportunity for industrial user to reduce their power costs and improve the efficiency of power energy. Although many methods for Economic Load Dispatch (ELD) under TOU pricing in continuous process industry have been proposed, there is difficulty in batch-type process industry because of that the power load units are not fixed but closely related with production planning and scheduling. In this paper, for hot rolling, a typical batch-type and energy intensive process in steel industry, a multi-objective production scheduling optimization model for ELD is proposed under TOU pricing, in which the objective is to minimize the power costs on the premise of ensuring the product quality. A NSGA-II based production scheduling algorithm is proposed to generate Pareto-optimal solutions, and then the TOPSIS based multi-criteria decision-making is performed to recommend an optimal solution to facilitate filed operation. Experimental results on practical production data show that the proposed method cut down the power costs by creating load units corresponding to electricity price and shifting loads to avoid on-peak time periods.

**Keywords** Hot rolling scheduling · Batch scheduling · Time-of-use pricing · Multi-objective optimization

## 1 Introduction

Time-of-Use (TOU) electricity pricing, a practical demand response program that aimed to improve the peak load regulation ability of power grid, have been implemented by many power supplier now. It provide a huge opportunity for electricity users to implement Economic Load Dispatch (ELD), which means cutting down their electricity costs by reducing the power loads at on-peak periods or shifting the loads from on-peak to off-peak periods.

Optimizing power costs under TOU pricing can be quite different from optimizing energy consumption, the industrial users need to adjust their production scheduling according to time-dependent electricity tariffs to reduce their power costs. In recent years, the methods that taking advantages of TOU pricing to implement ELD for industrial users have become a hot spot area. Shrouf et al. [1] proposed a single machine scheduling problem model, in which each time period has an associated price, the objective of model is to minimize power costs while considering traditional scheduling performance measures. Fang et al. [2] also considered the job scheduling on a single machine to minimize the total power costs under TOU pricing and proposed the problem solving algorithms for uniform-speed and speed-scalable machine environments respectively. Mitra et al. [3] formulated a mixed integer linear programming model for continuous process industry that allows optimal production planning and provided an case study for the time horizon of one week and hourly changing electricity prices, furthermore they improved the

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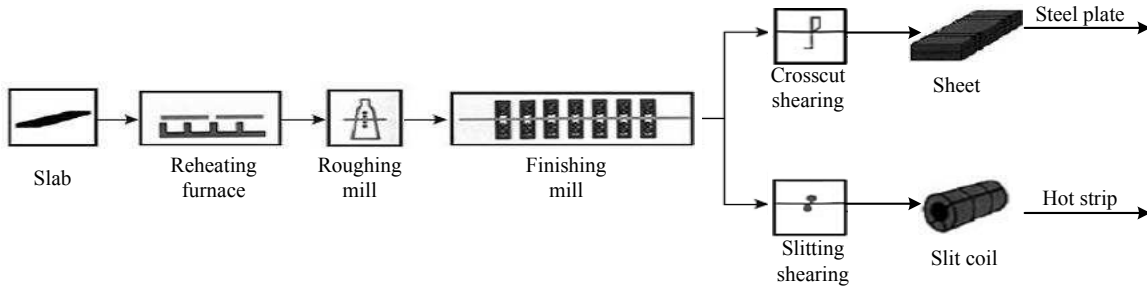
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**Fig. 1** Process flow of hot rolling production

model with integration of the operational and strategic decision-making [4]. Ashok [5] presented a theoretical model for batch-type load of process industry and proposed an integer programming method to reschedule their operations to reduce the electricity costs under time varying electricity tariffs, but what model he presented is an abstract theoretical model and difficult to be applied to production directly. Wang et al. [6] proposed an optimization model with both the consideration of power generation scheduling and batch production load scheduling in the iron and steel plant to minimize the electricity cost, which is convinced to be effective under TOU tariff, but in which the production load units in their model are determined by fixed production planning and scheduling, the results can not always be optimal.

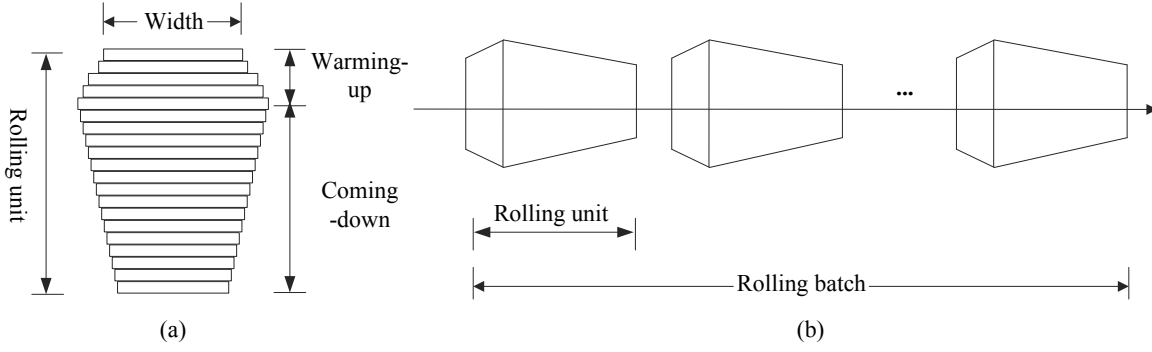
From above analysis, we know that there are great potential for ELD under TOU pricing based on production scheduling. Most of the existing literatures are focused on some specific problem in continuous process industry or the abstract problem model, for ELD in batch-type process industry, there is difficulty that the power load units are not fixed but closely related to the results of production planning and scheduling.

Hot rolling, a typical batch-type and energy intensive process in steel production with the characteristics of strong schedulability, has become an important aspect on production organization and energy saving in steel industry [7]. In general, the basic process flow of hot rolling production can be illustrated as Fig. 1. Hot rolling is mainly organized and carried out by the batch scheduling program in steel mill, the primary task of the program is arranging and sequencing slabs into rolling units with the objective of smoothing the jumps among adjacent slabs in width, gauge, and hardness, which will directly affect the product quality. Hot rolling production scheduling has attracted attention from academics and industry areas for a long time. The earlier proposed method by Kosiba et al. thought the steel production scheduling as a discrete event sequencing problem, and formulated it as traveling salesman problem [8]. Lopez

et al. [9] modeled the problem as a generalization of the prize collecting traveling salesman problem with multiple and conflicting objectives and constraints, and proposed a heuristic tabu search method to determine good approximate solutions. Tang and Wang [10] proposed a modified genetic algorithm to solved the problem based on the multiple travelling salesman problem model. Chen et al. [11] formulated a nonlinear integer programming model and Kim [12] gave some correction to the model, furthermore, Alidaee and Wang [13] proposed a corrected integer programming formulation and reduced the variables needed in the model. For most of the models are single objective or transformed models based on the weighted-sum approach, Jia et al. [14] formulated the problem as a multi-objective vehicle routing problem with double time windows model and proposed a decomposition-based hierarchical optimization algorithm to solve the model. Soon after, he proposed a P-MMAS algorithm to solve the problem, multi-criteria decision-making is performed to recommend the optimal solution from the Pareto-optimal solutions [15].

Due to high energy consumption and rising energy costs in hot rolling production [16], energy saving has also been considered except the traditional objective mentioned above. As shown in Fig. 1, the steel slabs need to be heated to high temperature before being rolled, the total energy consumption for heating is affected by the batch scheduling. Considering that high Direct Hot Charge Ratio (DHCR) in steel production has significant benefits such as energy cost reduction, some scholars made great efforts to improve the DHCR of slabs while performing batch scheduling [17,18]. Besides that, optimization of rolling schedule by adjusting reduction ratio of slabs between the rolling passes, another way to reduce the power consumption that used to drive the rolling motor, is also proposed [19,20,21] and has taken a certain effect.

As mentioned above, almost all the existed methods for hot rolling production scheduling are concentrated on internal production organization. Although some technical means have been proposed and applied



**Fig. 2** diagrammatic sketch of batch scheduling: (a) rolling unit, (b) rolling batch.

to achieve energy saving in some studies, the potential would have been exhausted because of the restriction of equipment and technology constraints. In this context, the methods that utilizing the favorable external environment should be explored for energy saving. TOU pricing provides an opportunity to reduce their power costs, but until now the papers to implement ELD under TOU pricing for hot rolling production are few published.

This paper consider the Hot Rolling Production Scheduling Problem (HRPSP) as a mixture of batch scheduling problem and time-dependant job-shop scheduling problem, the rolling units, which are seem as power load units in this paper, are planed and scheduled according to the TOU electricity prices. The primary objective of this model is to minimize the power costs while considering the traditional objective of minimizing penalties that caused by jumps between adjacent slabs. A multi-objective optimization model and algorithm are proposed to solve the problem, and a multi-criteria decision-making is performed to recommend an optimal solution to facilitate field operation.

## 2 Problem description and formulation

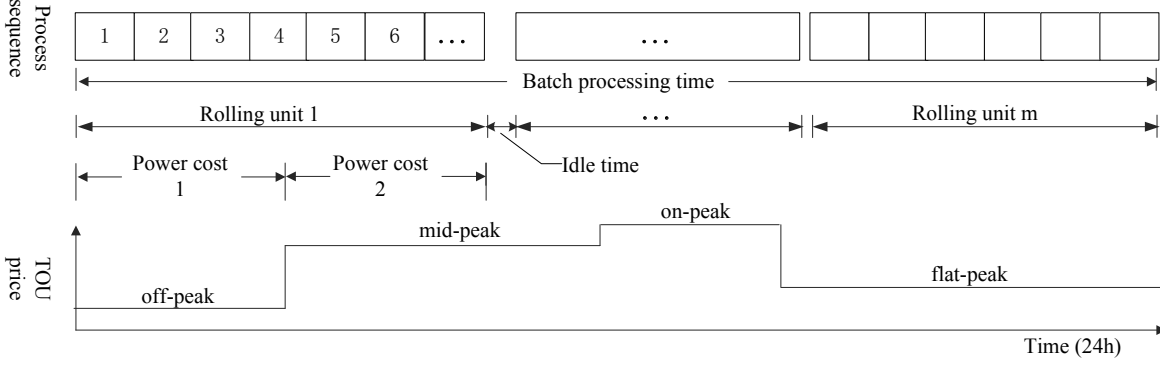
HRPSP is an extremely complex problem that has been convinced to have significant influence on product quality, production efficiency and energy consumption. In this paper, we consider the HRPSP as Hot Rolling Batch Scheduling Problem (HRBSP) combined with Job-shop Scheduling Problem (JSP), in which the HRBSP is focused on how the rolling units be scheduled and the JSP is concentrated on how and when the rolling units be produced.

### 2.1 Problem description

Hot rolling batch scheduling is a key process in hot rolling production. In this paper, we follow the scenario as shown in Fig. 2 to describe it as follows: The task of HRBSP is selecting, grouping and sequencing candidate slabs into rolling units under the constraints of production capacity and rolling rules. As shown in Fig. 2, a rolling unit that has a coffin-shaped width profile consists of a warming-up section and a coming-down section. In the previous one the slabs are arranged from narrow to wide to warm up the rolls, and in the later one the slabs are scheduled from wide to narrow to avoid marks on the surface of the coils. The major part of a rolling unit is the coming-down section, and the quality of rolling unit mainly depends on the sequence of slabs in it. In most cases, the warming-up section is just a minor part that can be determined manually.

There are several constraints that restrict the rolling batch scheduling, in which the most important one is smoothing jumps between adjacent slabs in width, gauge, and hardness. Other constraints, such as the cumulative rolling length of slabs in a rolling unit, the continuously rolled length of slabs with the same width and so on, should also be considered to ensure the product quality and production capability. For hot rolling is an key energy intensive process in steel industry, many approaches, such as optimization of batch scheduling with the objective of improving hot charging ratio and optimization of reduction schedule to reduce power energy for driving the rolling motor, have been proposed to achieve energy saving. In smart grid, TOU electricity pricing, which have been one of the most common implementation of demand response programs [22], provide a new opportunity for the steel mill to achieve ELD in hot rolling production, which means cutting costs by shifting loads among different time periods.

As shown in Fig. 3, a whole day is split into four types of time periods: on-peak, mid-peak, flat-peak and



**Fig. 3** Relationship between production scheduling and power costs under TOU pricing

off-peak periods, different types of time periods are associated with different electricity prices. We can also see that the power costs for each rolling unit, which are not only determined by the quantity of the power demand for each slab but also depended on the corresponding TOU electricity pricing, should be accumulated piecewise during the processing time.

Compared to the flat electricity pricing, the objective of ELD is to minimize the total power costs, which consists of the charges for power consumed and the additional operating costs due to shifting of loads, if any. In this paper, we assume that the rolling units can be scheduled freely, and there are no additional operating costs due to shifting of loads. In this context, the rolling production arranged in off-peak periods is encouraged and that in on-peak periods is deprecated to implement the peak load shifting. At the same time, the idle time should be allocated in the periods with high price as far as possible. In addition, we should know that the scheduling on fixed jobs are not always optimal, so the scheduled jobs, which means the rolling units generated by hot rolling batch scheduling, should be created with association to their operation time.

Finally, the problem is turned into optimal scheduling for minimizing the power costs that determined by batch scheduling solution and the job-shop scheduling solution under specified electricity pricing, while the traditional objective that smoothing the changes between adjacent slabs should not be ignored to ensuring the product quality.

## 2.2 mathematical formulation

As in literature [15], the basic model of the HRBSP can be interpreted as a Price Collecting Vehicle Routing Problem (PCVRP), which is an extension of VRP that well-known as an NP-hard problem. In the PCVRP model, it can be considered that each rolling unit is

a vehicle within limited capacity and each slab is a customer that can be visited at most once. Suppose that there are  $n$  slabs to be scheduled into  $m$  rolling units, the objective of the problem is to determine  $m$  routes (rolling units) to minimize the total distance traveled (penalties caused by jumps between adjacent slabs).

Considering the relationship between the slab processing sequence and the power costs under TOU pricing as shown in Fig. 3, the HRPSP can be formulated as a variant of PCVRP, in which the notations are defined as follows:

$T$ : the sequence of time periods,  $T = \{1, 2, \dots, t\}$ , where  $t$  is the number of time periods, a corresponding electricity price is associated to each period;

$\lambda_i$ : the duration of the  $i$ th time period, where  $i \in T$ ;

$\pi_i$ : the electricity price at the  $i$ th time period, where  $i \in T$ ;

$N$ : the sequence of all candidate slabs in the slab pool,  $N = \{1, 2, \dots, n\}$ , where  $n$  denotes the quantity of slabs;

$M$ : the sequence of rolling units,  $M = \{1, 2, \dots, m\}$ , where variable  $m$  is the quantity of rolling units;

$P_{ij}$ : the penalties to roll the  $j$ th slab immediately after the  $i$ th slab, where  $P_{ij} = p_{ij}^w + p_{ij}^g + p_{ij}^h$ , and the variables  $p_{ij}^w$ ,  $p_{ij}^g$  and  $p_{ij}^h$  respectively represent the penalties that caused by jumps from slab  $i$  moving to slab  $j$  in width, gauge, and hardness;

$l_i$ : the length of the  $i$ th slab, where  $i \in N$ ;

$w_i$ : the width of the  $i$ th slab, where  $i \in N$ ;

$L$ : the lower bound of the cumulative length of slabs that scheduled in a single rolling unit;

$U$ : the upper bound of the cumulative length of slabs that scheduled in a single rolling unit;

$R$ : the upper bound of the length of continuously rolled slabs with the same width in a single rolling unit;

$W_i$ : the power consumption of the  $i$ th slab during rolling process, where  $i \in N$ ;

$p_i$ : the processing time needed for the  $i$ th slab, where  $i \in N$ ;

$v_i$ : the idle time allocated to the  $i$ th rolling unit before the production, where  $i \in M$ ;

$TS$ : the total time that can be allocated for production.

To identify how the slabs are scheduled in rolling units and when the slabs are processed, four binary decision variables are defined as

$$y_i^k = \begin{cases} 1 & \text{if } i \vdash j, \\ 0 & \text{otherwise.} \end{cases}$$

where  $i \vdash j$  means the  $i$ th slab is scheduled in the  $j$ th rolling unit;

$$x_{ij}^k = \begin{cases} 1 & \text{if } y_i^k = 1, y_j^k = 1 \text{ and } j \succ i, \\ 0 & \text{otherwise.} \end{cases}$$

where  $j \succ i$  means the  $j$ th slab is rolled immediately after the  $i$ th slab;

$$s_{ij}^k = \begin{cases} 1 & \text{if } x_{ij}^k = 1 \text{ and } w_i = w_j, \\ 0 & \text{otherwise.} \end{cases}$$

which means the  $j$ th slab is rolled immediately after the  $i$ th slab in the  $k$ th rolling unit with the same width;

$$d_i^j = \begin{cases} 1 & \text{if } \sum_{\alpha < j} \lambda_\alpha \leq p_i' \leq \sum_{\alpha \leq j} \lambda_\alpha, \\ 0 & \text{otherwise.} \end{cases}$$

where  $\sum_{Ineq1} \lambda_\alpha$  means the duration of time periods subjected to the inequality *Ineq1*,  $p_i'$  means the processing start time for the  $i$ th slab, which is not only determined by decision variables  $x_i^j$  and  $y_i^j$ , but also depended on the processing time of rolling units, the idle time allocated for rolling units and duration of TOU time periods. Furthermore,  $p_i'$  can be expressed as

$$p_i' = \sum_{k < \delta} \sum_{\alpha \in N} y_\alpha^k \cdot p_\alpha + \sum_{\alpha \leq \delta} v_\alpha + \sum_{k \leq \theta(i)} \sum_{\alpha \in N} X_{\alpha\theta(k)}^\delta \cdot p_\alpha,$$

where the variable  $\delta$  represents the sequence number of rolling unit that the  $i$ th slab is scheduled,  $\theta(i)$  is a simple index function to return the position of the  $i$ th slab in the rolling unit,  $\sum_{\alpha \in N} y_\alpha^k \cdot p_\alpha$  is the cumulative processing time before the  $\delta$ th rolling unit,  $\sum_{\alpha \leq \delta} v_\alpha$  is the cumulative idle time allocated before the  $i$ th rolling unit,  $\sum_{k \leq \theta(i)} \sum_{\alpha \in N} X_{\alpha\theta(k)}^\delta \cdot p_\alpha$  represents the cumulative processing time before the  $i$ th slab in the  $\delta$ th rolling unit.

Based on the above notations and decision variables, a multi-objective hot rolling production optimization model can be formulated as

$$\min f_1 = \sum_{k \in T} \left( \pi_k \cdot \sum_{i \in N} \sum_{j \in T} W_i \cdot d_i^j \right) \quad (1)$$

$$\min f_2 = \sum_{k \in M} \sum_{i \in N} \sum_{j \in N} (P_{ij} \cdot x_{ij}^k) \quad (2)$$

subject to

$$\sum_{i \in N} x_{ij}^k = y_i^k, j \in N, k \in M \quad (3)$$

$$\sum_{j \in N} x_{ij}^k = y_i^k, i \in N, k \in M \quad (4)$$

$$\sum_{k \in M} y_i^k = 1, i \in N \quad (5)$$

$$\sum_{j \in N} s_{ij}^k \cdot y_j^k \cdot l_j \leq R, i \in N, k \in M \quad (6)$$

$$L \leq \sum_{i \in N} y_i^k \cdot l_i \leq U, k \in M \quad (7)$$

$$\sum_{i \in M} \leq TS - \sum_{k \in M} \sum_{i \in N} y_i^k \cdot p_i \quad (8)$$

where the objective (1) means to minimize the total power costs in hot rolling production to implement economic load dispatch, and the objective (2) is the traditional objective to ensure the product quality, which means to minimize the total penalties caused by jumps between adjacent slabs. Constraints (3) and (4) specify the sequence of slabs in a rolling unit. Constraint (5) ensures that each slab can be scheduled only once. Constraints (6) restrict the cumulative length of continuously rolled slabs with the same width in each rolling unit. Constraint (7) is the constraint of rolling mill production capacity, which restricts the lower and upper bounds of cumulative length of slabs in each rolling unit. Constraint (8) means that the total idle time allocated for rolling units cannot be greater than the margin of production capability.

### 3 Multi-objective production scheduling method

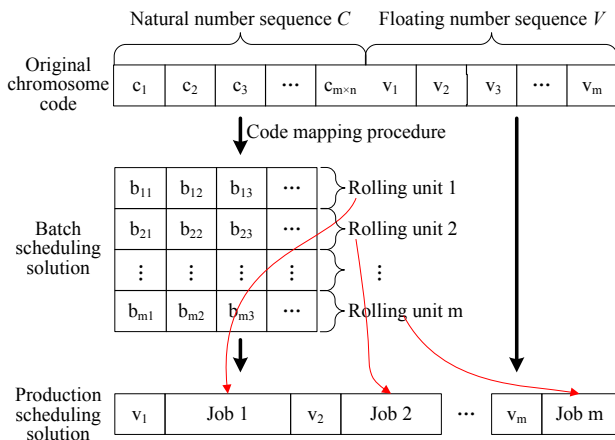
In this paper, the production scheduling method consists of two stages to implement ELD. In the first stage, objectives shown in Eq. (2) are optimized simultaneously, and then a set of Pareto-optimal solutions is generated by the multi-objective optimization algorithm. In the second stage, the TOPSIS based multi-criteria decision-making is performed to recommend an optimal solution to facilitate field operation.

### 3.1 Multi-objective production scheduling algorithm

NSGA-II is a classical multi-objective evolutionary optimization algorithm that proposed in 2000 by Deb et al. [23]. Because of the advantages of low computational expenditure, elitist strategy and less parameter sharing approach, NSGA-II has been widely used to solve the combinatorial optimization problems in engineering, such as the hydro-thermal power scheduling problem [24], job sequencing problem [25] and flow-shop scheduling problem [26].

In this paper, a NSGA-II based Multi-objective Production Scheduling Algorithm (MOPSA) is developed to solve the HRPSP, and the overall procedure of MOPSA is inherited from NSGA-II. Some personalized changes are made to instantiate the algorithm, in which the most import things are designing customized chromosome code and genetic operators to adapt specific problem.

For containing information about the batch scheduling and job-shop scheduling, a hybrid chromosome code consists of two parts as shown in Fig. 4 is designed. The first part is a natural number sequence  $C$  that can be transformed to a two-dimensional matrix  $B$  through the code mapping procedure, the matrix  $B$  represents a batch scheduling solution and the element  $b_{ij}$  in  $B$  is the original sequence of the  $j$ th slab in the  $i$ th rolling unit. For each  $i$ , if the minimal of  $j$  is found while  $b_{ij} = 0$ , it can be resolved that the last slab in the  $i$ th rolling unit is  $b_{i,j-1}$ . The second part is a floating number sequence  $V$  representing the idle time allocated for job-shop scheduling, where job means the production of rolling units.



**Fig. 4** Relationship between production scheduling and power costs under TOU pricing

According to above description, the hybrid chromosome code  $G$  can be formulated as

$$\begin{cases} G = (C, V) \\ C = (c_1, c_2, \dots, c_{m \times n}) , \\ V = (v_1, v_2, \dots, v_m) \end{cases}$$

where the element  $c_i$  in  $C$  is a natural number that ranged from 1 to  $m \times n$ ,  $m$  is the quantity of rolling units and  $n$  is the quantity of slabs to be scheduled, any two number  $c_i$  and  $c_j$  are associated with different values,  $v_i$  in  $V$  represents the idle time allocated to the  $i$ th rolling unit before rolling production.

Detailed steps of the code mapping procedure as mentioned above are listed as follows:

*Step 1:* Set  $f_i (i = 1, 2, \dots, n)$  to 0, where  $f_i$  is a flag and  $f_i = 1$  represent the  $i$ th slab has been arranged into rolling units; for the  $k$ th ( $k = 1, 2, \dots, m$ ) rolling unit, set  $num_k = 0$ , which means that the quantity of slabs in it is 0, set  $d_k = 0$ , which means that the accumulative rolling length of it is 0, set  $q_k = 0$ , which means that the continuously rolled length of slabs with same width is 0; set a loop variable  $j = 1$ ;

*Step 2:* Confirm the variables  $s$  and  $k$  that corresponding to the natural number  $c_j$ , by which it can be determined that the  $s$ th slab are scheduled in the  $k$ th rolling unit, where  $s$  and  $k$  can be calculated by

$$s = c_j - \left\lfloor \frac{c_j - 1}{m} \right\rfloor \times m$$

and

$$k = c_j - \left\lfloor \frac{c_j - 1}{n} \right\rfloor + 1.$$

*Step 3:* Check if the condition  $f_s = 0$  is satisfied:

- i. If satisfied, it means that the  $s$ th slab is an unscheduled slab. Then if  $w_s \neq w'_k$ , set  $q_k = 0$ , where  $w_s$  is the width of the  $s$ th slab and  $w'_k$  is the width of the latest appended slab in the  $k$ th rolling unit. Furthermore, if  $d_k + l_s \leq L$  and  $q_k + l_s \leq R$ , put the  $s$ th slab into the  $k$ th rolling unit, set  $num_k = num_k + 1$ ,  $d_k = d_k + l_s$ ,  $q_k = q_k + l_s$  and  $f_s = 1$ , the matrix  $B (= [b_{ij}])$  that represents slabs in rolling units will be updated as  $b_{k, num_k} = s$ ;
- ii. Otherwise, go to step 4;

*Step 4:* Update variable  $j = j + 1$ , go to step 2 to repeat the above operations until  $j = m \times n + 1$ ;

*Step 5:* Check if all the  $f_i = 0 (i = 1, 2, \dots, n)$  are satisfied:

i. If satisfied, it means that all the slabs are all allocated to rolling units with the constraints, and the matrix  $B (= [b_{ij}])$  represents a feasible solution of rolling batch; ii. Otherwise, a large number should be assigned to the fitness function  $f_1$  to avoid the chromosome be selected into the new population in the next selection operator.

We know that the constrained optimization is usually more difficult than the unconstrained optimization. The benefits of hybrid encoding and mapping procedure in this paper are not only containing complete information of the production scheduling but also handling the constraints. From step 5, it can be seen that all the constraints from Eq. (3) to (7) in Sec. 2.2 are satisfied in the finally accepted feasible solution, which is helpful to reduce the difficulty for problem solving.

Then in order to instantiate the MOPSA algorithm, customized genetic operators should be defined to match the hybrid chromosome code. For genetic algorithm, the most important genetic operators are selection, crossover, and mutation, which are briefly introduced as below. Selection operator, selecting individuals from population, is done based on the front rank of individuals by non-dominated sorting. For individuals with the same rank, which have the larger crowded distances will be adopted preferentially. In order to generate offspring from parent chromosomes, Partially Mapped Crossover (PMX) [27] and Scramble Sub-list Mutation (SSM) [28] are adopted to operate on the part of  $C$  in the chromosome code. The PMX operator is performed on two parent chromosomes: randomly select two crossover points  $k_1$  and  $k_2$ , swap the gene codes in the range  $[k_1, k_2]$ , after that, replace the other gene codes out of the range  $[k_1, k_2]$  according to the mapping relationship that determined by the middle section. Unlike the crossover operator, the SSM operator is performed on single parent chromosome: randomly select two positions  $p_1$  and  $p_2$  that separated less than a fixed length in the chromosome code, and then rearrange the gene codes between  $[p_1, p_2]$ . After crossover or mutation operators, the update on the part of  $V$  in chromosome code, which means idle time allocation for rolling units, should be performed immediately.

The detailed steps of idle time allocation as mentioned above for rolling units based on the part of  $C$  in the chromosome code are listed as follows:

*Step 1:* Confirm the electricity price  $\pi_i^s$  while the production started and price  $\pi_i^e$  while the production completed for the  $i$ th rolling unit, where  $i \in M$ ;

*Step 2:* Create a random floating number sequence  $V$  that represents the idle time allocated for rolling units before production. For elements  $V$ , the constraint as Eq. (8) in Sect. 2.2 must be satisfied;

*Step 3:* Sort the time periods in descending order based on the electricity price, after that a new set of time periods  $T' = (t'_1, t'_2, \dots, t'_t)$  is generated, in which the price associated with  $t'_k$  is  $\pi'_k$ ; define a loop variable  $j$  and set  $j = 1$ ;

*Step 4:* Adjust the idle time allocation for rolling units. Suppose that the  $i$ th rolling unit is started from time period  $t'_j$ , if  $\pi_i^e < \pi'_j$  and  $v_{i+1} > 0$ , then set  $v_{i+1} = 0$ ,  $v_i = v_i + v_{i+1}$ ; suppose that the  $i$ th rolling unit is completed in time period  $t'_j$ , if  $\pi_i^s < \pi'_j$  and  $v_i > 0$ , then set  $v_i = 0$ ,  $v_{i+1} = v_{i+1} + v_i$ ;

*Step 5:* Update variable  $j = j + 1$ , go to step 4 to repeat the operation for the left time periods until  $j = t$ , which represent the adjustment of idle time allocation is completed.

### 3.2 TOPSIS based multi-criteria decision-making

As MOPSA generate more than one Pareto-optimal solution, in order to facilitate field operation, only a few solutions should be accepted. In this paper, Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) [29], a widely used multi-criteria decision-making method to identify solutions from finite alternatives, is adopted as the method to select a recommended optimal solution.

Detailed steps of the TOPSIS based multi-criteria decision-making for HRPSP are listed as follows:

*Step 1:* The decision matrix  $X$  can be expressed as

$$X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ \vdots & \vdots \\ x_{m1} & x_{m2} \end{bmatrix},$$

where  $X$  is a two dimensional matrix with the size of  $m \times n$ , which means that there are  $m$  solutions generated by the multi-objective algorithm and two objectives for the HRPSP, element  $x_{ij}$  in  $X$  is the value of the  $j$ th objective with respect to the  $i$ th solution. Then the

normalized decision matrix  $Z(= [z_{ij}])$  can be calculated according to

$$z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^n x_{ij}^2}},$$

*Step 2:* Multiply the normalized decision matrix by its associated weights to calculate the weighted normalized decision matrix  $V(= [v_{ij}])$ , in which  $v_{ij}$  is calculated as

$$v_{ij} = w_j \cdot z_{ij},$$

where  $w_j$  is the weight associated with the  $j$ th objective, and in this paper,  $w_1$  and  $w_2$  are set to 0.4 and 0.6 respectively with the preference of the two objectives.

*Step 3:* Identify the the ideal solution  $s^+$  and the nadir solution  $s^-$  of each objective according to the following equations:

$$s^+ = (s_1^+, s_2^+),$$

$$s_j^+ = \begin{cases} \max_{1 \leq i \leq m} v_{ij} & \text{if } f_j \text{ is benefit-oriented,} \\ \min_{1 \leq i \leq m} v_{ij} & \text{if } f_j \text{ is cost-oriented.} \end{cases}$$

$$s^- = (s_1^-, s_2^-),$$

$$s_j^- = \begin{cases} \min_{1 \leq i \leq m} v_{ij} & \text{if } f_j \text{ is benefit-oriented,} \\ \max_{1 \leq i \leq m} v_{ij} & \text{if } f_j \text{ is cost-oriented.} \end{cases}$$

It should be known that both of the objectives in HRPSP are cost-oriented, which is said to find the minimum of objective functions.

*Step 4:* Measure the distance  $d_i^+$  and  $d_i^-$  of the  $i$ th solution from the ideal solution  $s^+$  and the nadir solution  $s^-$  by

$$d_i^+ = \sqrt{\sum_{j=1}^2 (v_{ij} - s_j^+)^2}, i = 1, 2, \dots, m,$$

$$d_i^- = \sqrt{\sum_{j=1}^2 (v_{ij} - s_j^-)^2}, i = 1, 2, \dots, m.$$

*Step 5:* Calculate  $C_i^*$  that represents the relative closeness of  $i$ th solution with respect to the ideal solution according to

$$C_i^* = \frac{d_i^-}{(d_i^- + d_i^+)}, i = 1, 2, \dots, m.$$

After completing the above steps, the decision-making can be finally performed on the Pareto-optimal solutions according to the sequence that determined by  $C_i^* (i = 1, 2, \dots, m)$  in descending order, which solution that owns the maximal relative closeness will be selected as the recommended optimal solution.

## 4 Experimental procedure and results

To evaluate the effectiveness and advantages of the proposed method, four groups of production data with different slab quantities and processing time are collected from Xiangtan Steel Mill to be used in the experiment. As shown in Table 1, each group of data represents a different characteristic. For the production data, many varieties means that there are numerous variety of width, gauge and hardness for the slabs, and the penalty between adjacent slabs will be larger, at the same time, full load means that the slab processing will take long time and the idle time to allocate for rolling units will be fewer.

The data in Table 2 is the TOU electricity tariffs that actually performed in the steel mill. According to daily power load distribution, the whole day is split into eight periods that covered four types of time periods, and for each type of time period, a corresponding electricity price is assigned.

According to constraints of equipment and production capability, the lower and upper bound of the cumulative length of slabs that scheduled in a single rolling unit are respectively set to 5 and 10 kilometers, and the upper bound of the continuously rolled length of slabs with same width is set to 1 kilometers. For a specific slab, the rolling length, processing time and power consumption can be obtained by the hot rolling process control system in steel mill. The penalties that caused by jumps between adjacent slabs in width, gauge and hardness are adopted by referencing to [8].

To obtain excellent performance and preferable results, choosing suitable parameters values for the MOPSA

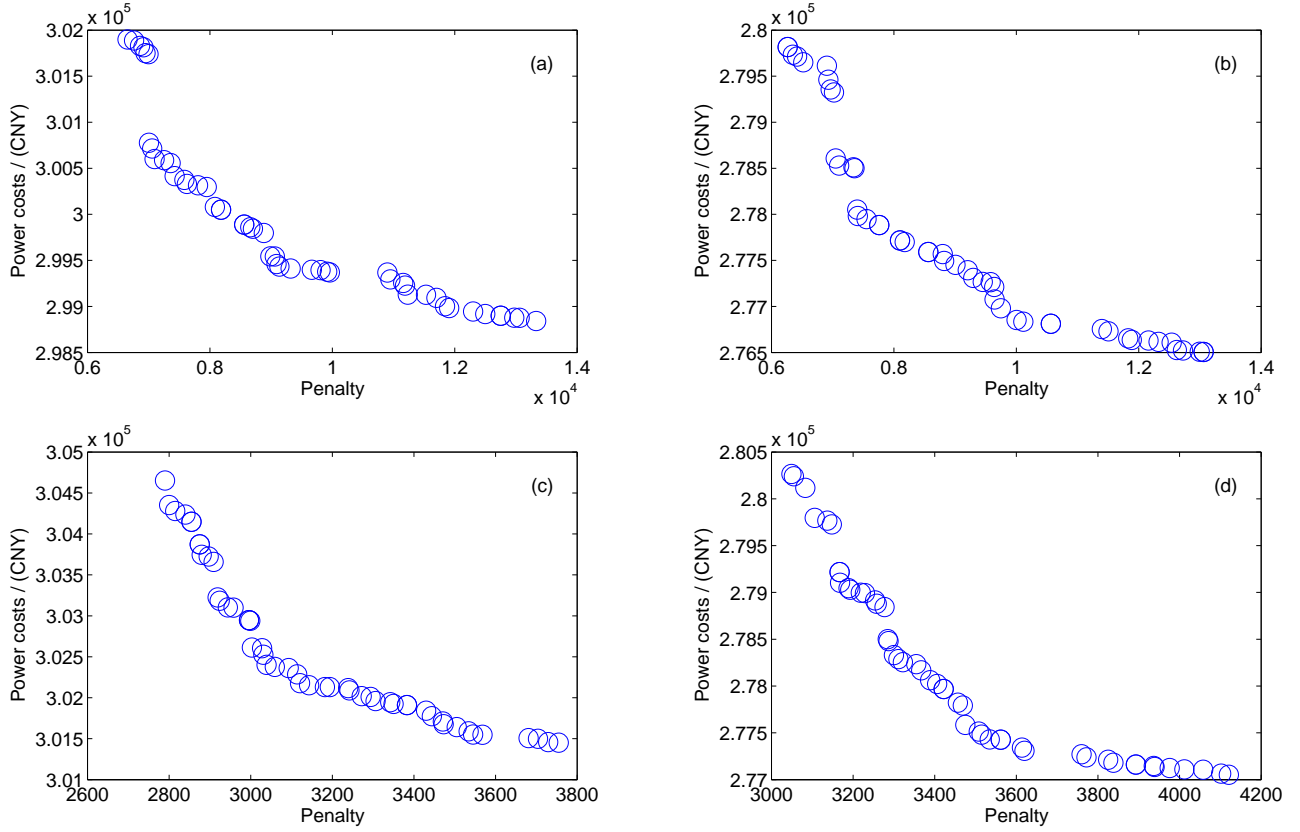
**Table 1** Production data description

Group Id	Slab quantity	Rolling units quantity	Characteristics
1	442	8	Many varieties, full load
2	418	8	Many varieties, not full load
3	441	8	Few varieties, full load
4	417	8	Few varieties, not full load

**Table 2** Time-of-use electricity tariffs

Time period	Time frame	Electricity price / (CNY · kWh <sup>-1</sup> )
on-peak	18:00-21:00	0.878
mid-peak	08:00-11:00, 15:00-18:00	0.778
flat-peak	07:00-08:00, 11:00-15:00, 1:00-22:00	0.628
off-peak	00:00-07:00, 22:00-24:00	0.428





**Fig. 5** Pareto fronts for different groups of production data: (a) group 1, (b) group 2, (c) group 3, (d) group 4.

algorithm is very important. Using parameter sensitivity analysis based on a lot of test, a group of empirical values is determined, in which the probability of crossover and mutation are set to 0.4 and 0.6 respectively, the size of population is set to 50, and the maximum iterations of algorithm is set to 2000.

The proposed method is implemented in MATLAB 7.0, and the experimental results are obtained on a PC with Pentium(R) 2.13 GHz dual core processor and 2 GB memory.

#### 4.1 Experimental results

To measure the effectiveness of the proposed method, the graphical approximate Pareto fronts returned by MOPSA are illustrated in Fig. 5, in which the sub diagrams (a) to (d) are related to four groups of production data respectively. In each sub diagram, the horizontal axis denotes the total penalties caused by jumps and the vertical axis denotes the power costs in production. Because both of the objectives in our problem are cost-oriented, we know that the smaller of objectives, the better performance of the algorithm will be.

As shown in Fig. 5, we can see that for each group of production data, a certain number of Pareto-optimal solutions are generated and uniformly scattered on the Pareto fronts, which means that the MOPSA has strong ability for searching optimal solution. For sub diagrams (a) and (b), the penalty scores are obvious larger than (c) and (d), that is because the varieties of slab in group 1 and 2 are varied and abundant. At the same time, the power costs in (a) and (c) are larger than (b) and (d), which is because that the production load in group 1 and 3 are fuller. Nevertheless, it should be noted that the true Pareto-optimal front is not known for the practical production data, so the fronts in Fig. 5 are all pseudo-optimal Pareto fronts constructed by combining all the non-dominated solutions and removing all the dominated solutions.

In order to facilitate field operation, the TOPSIS based multi-criteria decision-making is performed to select the only one solution from the Pareto-optimal solutions. As shown in Fig. 5, the dimension of two objectives are different, which means  $f_1$  is much bigger than  $f_2$  and  $f_1$  fluctuate in a smaller range than  $f_2$ , which will affect the practical importance of objectives in TOPSIS sorting. In order to put the values of two objectives into similar scale, the transformation for el-

**Table 3** Partial TOPSIS sorting results

Solution Id	Relative closeness	$f_1$ (/CNY)	$f_2$
S1	0.7772	300600.61	7097
S2	0.7758	300414.79	7424
S3	0.7458	300079.79	8081
S4	0.7418	300296.46	7949
S5	0.6845	301898.17	6654
S6	0.5627	299376.89	9916
S7	0.4514	299368.24	10899
S8	0.3155	298841.41	13330

**Table 4** Comparison of scheduling solutions

Group Id	Proposed method		Conventional method	
	Power cost/(CNY)	Penalty	Power cost/(CNY)	Penalty
1	300600.06	7097	309389.81	6737.00
2	278606.13	7049	297575.21	6265.00
3	302401.27	3039	310091.60	2765.00
4	278330.86	3300	299708.13	2923.00

ements in decision matrix is performed as

$$x_{ij} = x_{ij} - \min_{1 \leq i \leq m} x_{ij},$$

where  $x_{ij}$  means the value of the  $j$ th objective with respect to the  $i$ th solution, and variable  $m$  is quantity of Pareto-optimal solutions.

After the transformation, TOPSIS sorting is performed and partial results for the first group of production data are given in Table 3. According to the descending order based on relative closeness, the solution S1, which owns the maximal relative closeness 0.7772, is finally selected as the recommended optimal solution.

Comparisons of scheduling results by two methods are given in Table 4. The method in [15] is adopted as the conventional method, in which the penalty score is minimized by GA algorithm and the power costs is never taken into consideration. It can be seen clearly that the penalty by the proposed method increased than that by traditional method but the magnitude is very small for any group of data. Compared to reference value in [14, 15], the penalty is still accepted in engineering, and the product quality would not be affected obviously, but at the same time, much lower power costs are obtained by the proposed method with the same power consumption, which indicating that ELD, the primary objective of the proposed method in this paper, is implemented by utilizing the TOU pricing in electricity market environment.

#### 4.2 Analysis of scheduling results

To further confirm the effectiveness of the proposed method, the job scheduling results on the first group

of production data is chosen as a scenario to perform a more detailed analysis. Values of rolling parameters by the two method are both given in Table 5, from which we can see that the quantities of slabs in different rolling units are basically uniform, and the rolling length for each rolling unit is restricted to less than 10 kilometers. Besides that, some other parameters for each rolling unit, such as processing time, power demand, average power load, processing start time, processing complete time, and allocated idle time, are also given to describe the results of job scheduling, all of the above parameters represent that the proposed method is effective while owning the better performance on objectives  $f_1$  as shown in Table 4.

Considering rolling units as production jobs and illustrating them in a double longitudinal axis diagram as shown in Fig. 6, in which the sub diagrams (a) – (d) are corresponding to group 1 to 4 of production data respectively. The horizontal axis in sub diagram represents the rolling processing time, and the left and right longitudinal axes indicate the power load and electricity price respectively. The jobs scheduled by proposed method are represented by solid lines that started and ended with a circle, and the jobs scheduled by conventional method are illustrated by solid lines that started and ended with a square. From the figure we can identify the production load and the electricity price at any time.

As it can be seen, in any sub diagram, the heavy loads are created and mainly arranged in off-peak and flat-peak periods by the proposed method, while the light loads are arranged at on-peak or mid-peak periods. In addition, the idle time is allocated for jobs under constraints of production capability by the proposed method. For scenario in this paper, we can see that the idle time is allocated at 18:00 to 21:00, which is the obvious on-peak time period.

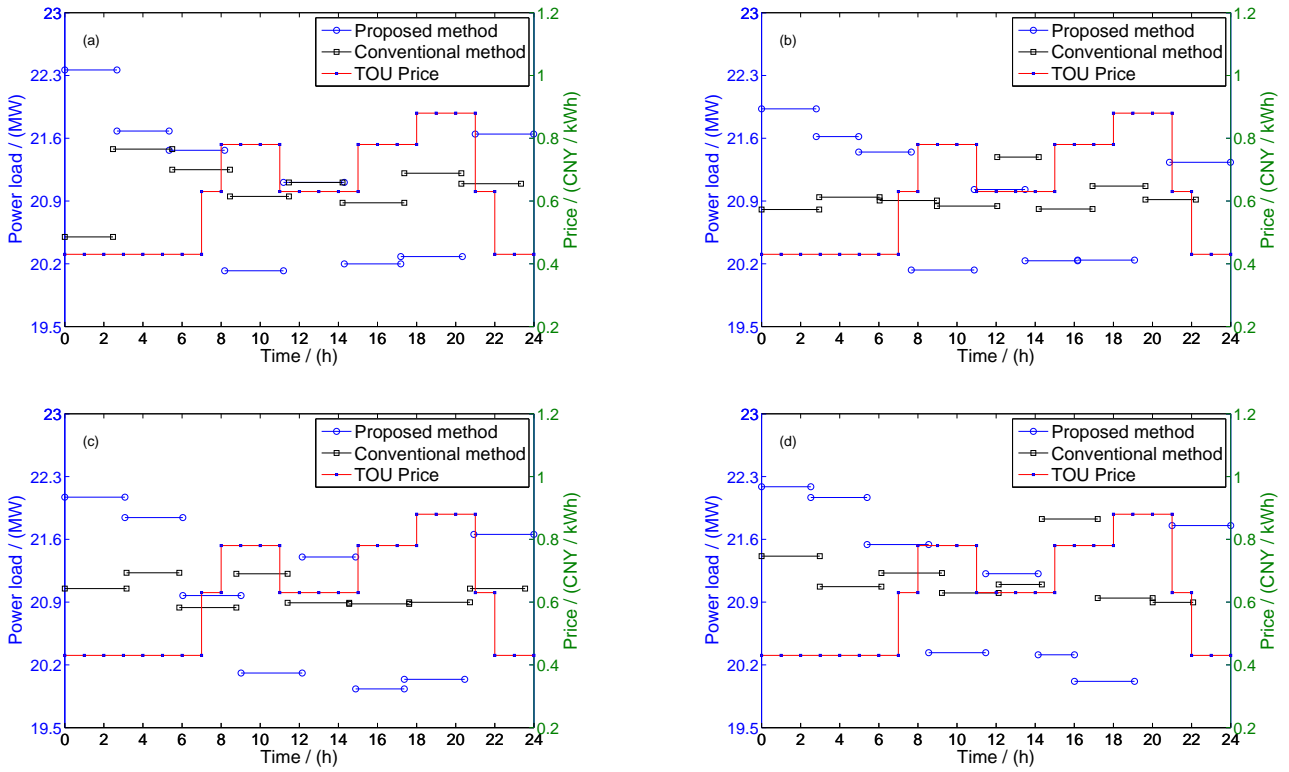
Form above analysis, we can know that the advantages of the proposed method can be attributed to two aspects, one is the TOU pricing based power load creating, which means the rolling units are not fixed but depended on their processing time and the corresponding electricity price, another one is load shifting, which means arranging the processing sequence and time for rolling units to avoid on-peak time periods.

#### 4.3 Discussion of peak load regulation

From perspective of power grid, the intention of implementing TOU electricity pricing is encouraging the power users to participate peak load regulation, which means reducing power loads in on-peak periods or shifting power loads from on-peak to off-peak periods, and

**Table 5** Detailed comparison of the scheduling solution for the 1st group of production data

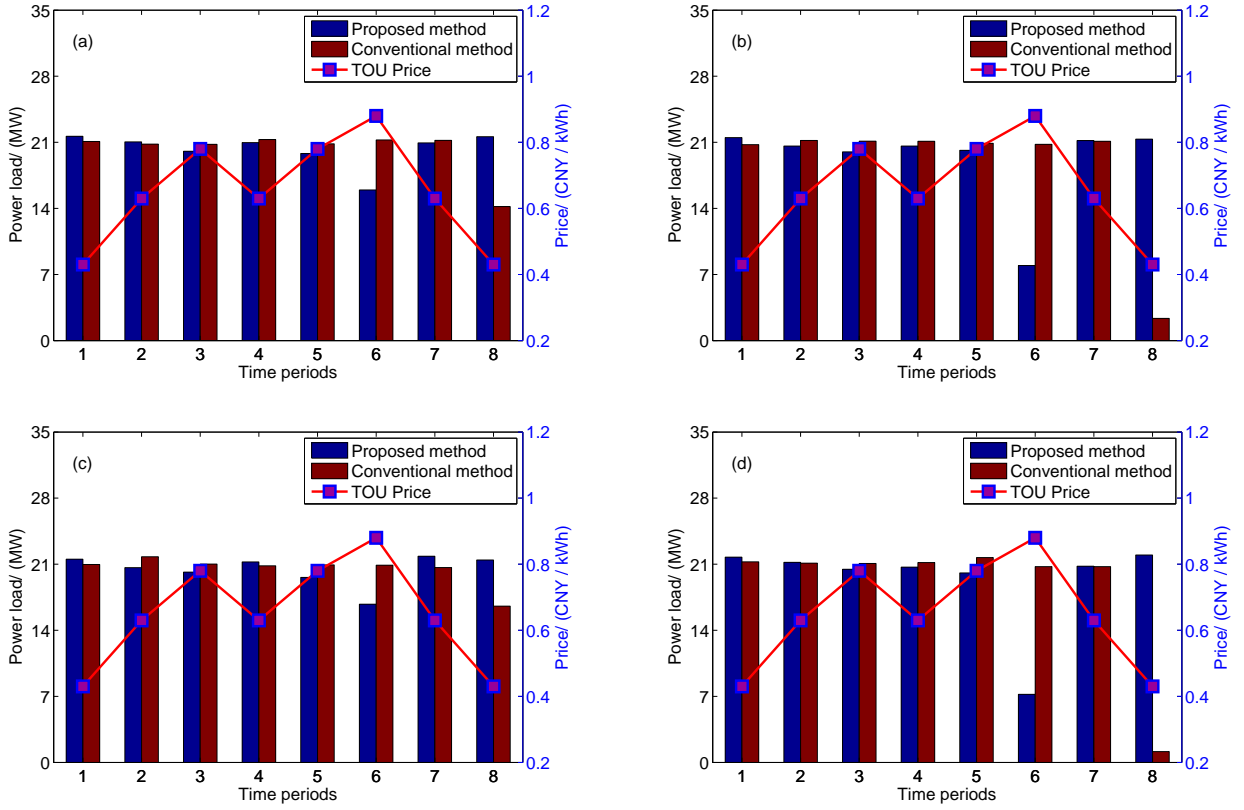
Scheduling method	Rolling unit No.	Slab quantity	Rolling length / (km)	Processing time / (h)	Power demand / (MW·h)	Average power load / (MW)	Processing start time	Processing complete time	Allocated idle time
Proposed method	1	59	9.12	2.67	59.71	22.36	00:00	02:40	0
	2	50	8.60	2.67	57.89	21.68	02:40	05:20	0
	3	51	9.97	2.84	60.96	21.47	05:20	08:10	0
	4	59	9.48	3.01	60.57	20.12	08:10	11:11	0
	5	57	9.97	3.11	65.64	21.11	11:11	14:18	0
	6	57	8.71	2.89	58.37	20.20	14:18	17:11	0
	7	50	9.98	3.15	63.88	20.28	17:11	20:20	0
	8	59	9.99	2.99	64.72	21.65	21:00	24:00	0.66
Conventional method	1	46	7.35	2.47	50.63	20.50	00:00	02:28	0
	2	58	9.94	3.03	65.08	21.48	02:28	05:30	0
	3	57	9.88	2.95	62.69	21.25	05:30	08:27	0
	4	57	9.98	3.02	63.27	20.95	08:27	11:28	0
	5	51	9.05	2.74	57.83	21.11	11:28	14:12	0
	6	60	9.73	3.16	65.98	20.88	14:12	17:22	0
	7	56	9.93	2.92	61.93	21.21	17:22	20:17	0
	8	57	9.94	3.05	64.33	21.09	20:17	23:20	0

**Fig. 6** Scheduling results on different groups of production data: (a) group 1, (b) group 2, (c) group 3, (d) group 4.

is good for promoting the stability of power grid. In this section, we try to discuss the effectiveness of the proposed method on peak load regulation.

The power load distribution among time periods by the two scheduling methods are provided in Fig. 7, in which the comparisons for all groups of production data are illustrated by the sub diagrams (a) – (d) respec-

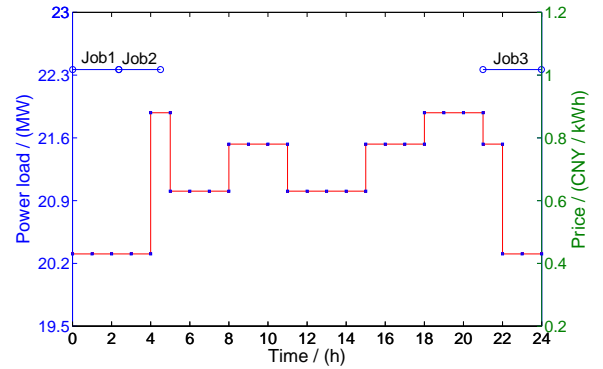
tively. As it can be seen, for any group of production data, the average power load is greatly reduced in the on-peak periods and substantially increased in the last off-peak period, at the same time, the power loads in the first off-peak period increased slightly. Especially, for group 2 and group 4, in which the production data are characterized by not full production load, the power load



**Fig. 7** Power load distribution among time periods for different groups of production data: (a) group 1, (b) group 2, (c) group 3, (d) group 4.

increased in the on-peak period and reduced in the last off-peak period significantly. Combined with Fig. 6, we can know that the above results are mainly attributed to power load shifting from on-peak to off-peak periods. From the results in Fig. 7 and the above analysis, it can be concluded preliminarily that the ELD in this paper promoted the peak load regulation ability of the power grid, and the effectiveness of peak load regulation is majorly affected by the full condition of production load. The idler of production load, the more potential of the peak load regulation ability will be.

Nevertheless, it should be noted that the ELD don't always promote the peak load regulation. Considering that a new TOU tariff, in which an on-peak period is closed to the off-peak period, is provides to users as shown in Fig. 8, and three jobs need to be completed. In order to minimize the power costs, the Job1 and Job3 are arranged as close as possible to the off-peak periods. In order to implement peak load regulation, the Job2 should be arranged in the mid-peak or flat-peak periods, but considering minimizing the power costs, the left Job2 would be arranged across the off-peak and on-peak period, in which most of the job processing time will be covered by the off-peak period. In this case,



**Fig. 8** Scheduling results under a new TOU tariffs

the power load in the first on-peak period will not fall but rise, which deviate from the original intention of TOU pricing. This is a special case that arising from the lack of interaction between the power supply side and the demand side in electricity market, and it needs to be solved with the participation and collaboration of both sides, which will not be discussed in depth in this paper. In our method, we just perform the optimal production scheduling according to the provided price signals to reduce the power costs.

## 5 Conclusions

This paper presented the challenge of energy saving in hot rolling production and formulated a multi-objective optimization model of HRPSP under TOU electricity pricing, in which the hot rolling batch scheduling and the time-dependant job-shop scheduling were both taken into consideration. Objective of the proposed model is to minimize the power costs while considering traditional objective for measuring the scheduling performance. A NSGA-II based multi-objective production scheduling algorithm was proposed to solve the problem, in which the chromosome code and genetic operators are custom-designed to match the HRPSP, and moreover, a TOPSIS based multi-criteria decision-making was performed to recommend an optimal solution from the Pareto-optimal solutions to facilitate filed operation. Experimental procedure on practical production data was given out, the results and analysis showed that the proposed method can reduce power costs in hot rolling production on the premise of ensuring the product quality. Considering multiple production lines existed in most steel mills, the further study on HRPSP integrated parallel machine job-shop scheduling will be the next work, which is expected to be more difficult but have greater benefits.

**Acknowledgements** Thanks are due to Prof. Jing Zhang and Dr. Chengqing Li for their assistance with the valuable suggestions and discussion. This work was supported by National Natural Science Foundation of China (61402391, 61170191), Natural Science Foundation of Hunan Province of China (14JJ2071), and Educational Commission of Hunan Province of China (11C1234).

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